

Fifth Semester B.E. Degree Examination, June / July 08
Modern Control Theory

Time: 3 hrs.

Max. Marks:100

Note : Answer any FIVE full questions.

- 1 a. Explain P, PI, PID controllers. (08 Marks)
 b. Construct the state model using phase -variables for the following equation and draw the state diagram:

$$\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 2y = 5u. \quad (12 \text{ Marks})$$

- 2 a. Explain the terms with an example:
 i) State ii) State variables iii) State vector iv) State space. (08 Marks)
 b. Obtain - i) First companion and ii) Second companion form, for the equation

$$G(s) = \frac{s+3}{s^3 + 9s^2 + 24s + 20}$$

Draw the signal flow graphs. (12 Marks)

- 3 a. Define state transition matrix. List out its properties. (08 Marks)
 b. Obtain state transition matrix by
 i) Laplace transformation and
 ii) Cayley-Hamilton theorem
 for the following system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (12 \text{ Marks})$$

- 4 a. Find the time-response of the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(t)$$

where $u(t)$ is the unit step occurring at $t = 0$ and $x^T(0) = [1 \ 0]$ (10 Marks)

- b. Obtain the transfer functions of the systems having the state models

$$i) \dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \quad \text{and} \quad Y = [1 \ 0]X$$

$$ii) \dot{x} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix} X + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u \quad \text{and} \quad Y = [1 \ 2]X \quad (10 \text{ Marks})$$

- 5 a. Explain the common physical non linearities. (10 Marks)
 b. What is the significance of phase-plane plot? Explain the various steps of plotting phase-plane trajectories by either phase-plane method or isocline method. (10 Marks)
- 6 a. What are singular points? Explain the different singular points with respect to stability of non-linear systems. (10 Marks)

- b. Consider the system described by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Using state feedback control $u = -kx$, it is desired to have the closed loop poles at $s = -1 \pm j2$, $s = -10$. Find the state feedback matrix using Ackermann's formula. (10 Marks)

- 7 a. Determine whether or not the following quadratic form is negative definite.

$$Q = -x_1^2 - 3x_2^2 - 11x_3^2 + 2x_1x_2 - 4x_2x_3 - 2x_1x_3 \quad (10 \text{ Marks})$$

- b. Obtain the Liapunov candidate for the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (10 \text{ Marks})$$

- 8 a. Using Krasovskii's theorem, find the stability region of the equilibrium state at $x = 0$ for the following

$$\dot{x}_1 = -x_1$$

$$\dot{x}_2 = x_1 - x_2 - x_2^3 \quad (10 \text{ Marks})$$

- b. Explain the concept of complete state controllability. Obtain the conditions on a, b and c for complete state controllability for

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 1 \\ 0 & 0 & \lambda_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} a \\ b \\ c \end{bmatrix} [u] \quad (10 \text{ Marks})$$
